STATISTICS (C) UNIT 1 **TEST PAPER 8**

1. A private-hire company has five taxis and eight minibuses. For a race meeting the company is asked to provide three taxis and four minibuses.
Find the number of different ways in which this can be done. [3]

2. Trainees who have recently started a word-processing course are assessed on their typing speed in words per minute and their accuracy, expressed as a percentage. On a particular day seven trainees were tested, with the following results :

Trainee	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Typing speed (w.p.m.)	30	54	22	52	46	50	49
Accuracy (%)	50	74	45	54	48	91	60

(i) Calculate Spearman's coefficient of rank correlation between the two qualities tested. [4]
Given that the product moment correlation coefficient between typing speed and accuracy was 0·591,
(ii) comment on the comparative values of the two correlation coefficients. [2]

3. A darts player throws two darts, attempting to score a bull's-eye with each. The probability that he will achieve this with his first dart is 0·25. If he misses with his first dart, the probability that he will also miss with his second dart is 0·7. The probability that he will miss with at least one dart is 0·9.
(i) Show that the probability that he succeeds with his first dart but misses with his second is 0·15. [3]
(ii) Find the conditional probability that he misses with both darts, given that he misses with at least one. [3]

4. A hand of 12 cards contains three 2s, four 3s, two 4s, two 5s and one 6. The random variable X represents the number on a card chosen at random from this hand.
(i) Draw up a table to show the probability distribution of X. [3]
(ii) Calculate E(X) and Var(X). [5]

5. (i) Find the mean and the standard deviation of the 20 observations of a variable X given by the following table:

x	75	80	85	90	95	100	105	110
Frequency	1	2	3	6	4	2	1	1

[5]

Another 20 observations of X had a mean, μ, of 87·5 and $\sum (x-\mu)^2 = 1750$.

(ii) Find the standard deviation of these 20 observations. [2]

(iii) Use your answers to compare the two sets of observations. [2]

6. In an orchard, all the trees are either apple or pear trees. There are four times as many apple trees as pear trees. Find the probability that, in a random sample of 10 trees, there are

(i) equal numbers of apple and pear trees, [3]

(ii) more than 7 apple trees. [2]

(iii) Find the expected number of pear trees in a sample of 60 trees in the orchard. [2]

(iv) Find the probability that exactly 15 out of a sample of 60 trees are pear trees. [3]

7. A missile was fired vertically upwards and its height above ground level, h metres, was found at various times t seconds after it was released. The results are given in the following table:

t	1	2	3	4	5	6	7
h	68	126	174	216	240	252	266

It is thought that this data can be fitted to the formula $h = pt - qt^2$.

(i) Show that this equation can be written as $\frac{h}{t} = p - qt$. [1]

(ii) Plot a scatter diagram of $\frac{h}{t}$ against t. [5]

Given that $\sum h = 1342$, $\sum \frac{h}{t} = 371$ and $\sum \frac{h^2}{t^2} = 20\,385$,

(iii) find the equation of the regression line of $\frac{h}{t}$ on t and hence write down the values of p and q. [6]

(iv) Use your equation to find the value of h when $t = 10$. Comment on the implication of your answer. [3]

(v) Find the product-moment correlation coefficient between $\frac{h}{t}$ and t and state the significance of its value. [3]

STATISTICS 1 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1. ${}^5C_3 \times {}^8C_4 = 10 \times 70 = 700$ — M1 A1 A1 — 3

2. (i)

Ranks :	2	7	1	6	3	5	4
	3	6	1	4	2	7	5
d	1	1	0	2	1	2	1

B1 B1

$\sum d^2 = 12$ $r_S = 1 - 72/336 = 0{\cdot}786$ — M1 A1

(ii) Fairly good rank correlation does not imply a high p.m.c.c. — B2 — 6

3. (i) Let P(miss after hit) = x $0{\cdot}75 + 0.25x = 0.9$ — M1

$x = 0{\cdot}6$ P(H, M) = $0{\cdot}25 \times 0{\cdot}6 = 0{\cdot}15$ — A1 A1

(ii) P[(M, M) | at least 1 miss] = $(0{\cdot}75 \times 0{\cdot}7) \div 0{\cdot}9 = 0{\cdot}583$ — M1 A1 A1 — 6

4. (i)

x	2	3	4	5	6
P($X = x$)	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

M1 A1 A1

(ii) E(X) = 3·5 $E(X^2) = 1 + 3 + \frac{8}{3} + \frac{25}{6} + 3 = 13{\cdot}8$ Var(X) = 1·58 — B1 M1 A1 M1 A1 — 8

5. (i) $\sum x = 1825$ $\sum x^2 = 167\,925$ — B1 B1

Mean = $1825 \div 20 = 91{\cdot}25$ s.d. = $\sqrt{69{\cdot}7} = 8{\cdot}35$ — M1 A1 A1

(ii) s.d. = $\sqrt{(1750/20)} = 9{\cdot}35$ — M1 A1

(iii) Second set had lower values overall and a wider spread — B1 B1 — 9

6. (i) Pears ~ B(10, 0·2) $P(X = 5) = 0{\cdot}9936 - 0{\cdot}9672 = 0{\cdot}0264$ — B1 M1 A1

(ii) $P(X < 3) = P(X \leq 2) = 0{\cdot}678$ — M1 A1

(iii) $E(X) = 60 \times 0{\cdot}2 = 12$ — M1 A1

(iv) In B(60, 0·2), $P(X = 15) = {}^{60}C_{15}(0{\cdot}2)^{15}(0{\cdot}8)^{45} = 0{\cdot}0759$ — M1 A1 A1 — 10

7. (i) $h = t(p - qt)$ $\frac{h}{t} = p - qt$ — B1

(ii)

t	1	2	3	4	5	6	7
$\frac{h}{t}$	68	64	58	54	48	42	38

M1 A1

Scatter graph drawn — B3

(iii) $\sum t = 28$, $\sum t^2 = 140$, $\sum t(\frac{h}{t}) = \sum h = 1342$ — B1 B1

$\frac{h}{t} - \frac{371}{7} = \frac{7(1342) - 28(370)}{7(140) - 28^2}(t - \frac{28}{7})$ $\frac{h}{t} - 53 = -5{\cdot}07(t - 4)$ — M1 A1

$\frac{h}{t} = -5{\cdot}07t + 73{\cdot}3$ $p = 73{\cdot}3$, $q = 5{\cdot}07$ — M1 A1

(iv) $t = 10$: $h/10 = 22{\cdot}6$ $h = 226$ Coming down again — M1 A1 A1

(v) $r = \dfrac{-994}{\sqrt{(7(20385) - 371^2)}} = -0{\cdot}999$ — M1 A1

Shows that the formula is a very good fit to the data and confirms that $\frac{h}{t}$ decreases as t increases. — A1 — 18